Introduction to Mathematical Thinking. Proof Evaluation Rubric (beta)

Evaluating mathematical proofs is a holistic activity that requires considerable experience to do well. In this course, we use proof evaluation to gain greater understanding of what constitutes a good proof. To assist you to do this, I've identified several factors of good proofs that a reviewer should look for. The rubric asks you to consider a proof in terms of six categories (listed in the left-hand column) and try to assign 0, 2, or 4 points for each category to the best of your ability. (You can enter 1 or 3 points if you find it hard deciding between the categories, but first try to assign 0, 2, or 4.) An argument that does not address the stated result (or answer the given question) should be given 0 marks in all categories.

Even when broken down this way, you will find you still have to make value judgments. To help you develop that ability, initial use of the rubric is accompanied by a tutorial lecture demonstrating the rubric's use.

* You may be familiar with using a rubric to assign grades to students, where rubric is used to achieve uniformity across different graders. ***We are using this rubric very differently***: Not to assign meaningful grades, but to assist you in assessing proofs from different perspectives.
* The final category (Overall valuation) is the grade you would assign to a student if you were grading their work. In this course, we are focusing separately on different aspects of proofs, and your evaluation will be assessed on the degree to which you cover all those features – not just the "final mark".
* Different courses for different students would put different weights on the first five categories. In this course, the weightings are all equal. This is both to keep the process as simple as possible and to reflect the fact that all five features are important in coming to understand proofs and their different uses.

	Novice (0 points)	**Apprentice** (2 points)	**Practitioner** (4 points)
Logical Correctness	The answer given is fundamentally wrong.	The approach is generally correct, but there is at least one significant error.	Other than perhaps a minor slip, the proof is complete and correct.
Clarity	Overall, the argument is hard or impossible to follow.	Can follow it with some effort. Some parts may be clearer than others.	Clear and easy to follow throughout.
Opening	No opening statement of what is being proved. No mention of use of standard method, where relevant (e.g. induction).	There is a statement of what is being proved (inc. mention of a standard method, if relevant), but it comes later and/or is incomplete.	Clear, correct opening statement of what is being proved, with statement of method if a standard method is used.
Stating the conclusion	Argument ends abruptly, without stating or acknowledging a conclusion.	Argument ends with some form of concluding statement, but it is not clear and definitive.	Argument concludes with a clear and concise statement indicating that the desired result has been established.
Reasons	Significant steps presented without justification.	Some significant steps are justified, but at least one is not.	Reasons are given for all significant steps.
Overall valuation	Overall, this is not a good answer.	The answer is fairly good, but there is room for improvement.	Discounting small, minor slips, this is a good answer.

USE OF THE RUBRIC IN THE TEST FLIGHT. If an evaluation rubric is used to assign meaningful student grades, it should be done by someone who is very familiar with mathematical proofs and with the particular topic, and in addition has experience grading. None of these is possible in a MOOC. For that reason alone, we would have to use rubric-guided evaluation *as a learning tool*, not to assign grades. Experience has shown, however, that by approaching calibrated peer evaluation with the mindset of *trying to provide an accurate, formative evaluation of someone's work*, both parties learn a lot. In particular, (1) the person whose work is being graded obtains *helpful, constructive* feedback about her or his work from others taking the same class, and (2) the act of trying to assign a numerical grade to someone else's work has been shown to yield considerable learning benefits for the grader. But remember, it takes any of us considerable courage to show our work to others, even anonymously, particularly if we are learning something for the first time and consequently are not confident in our ability. The most helpful mindset to adopt in evaluating someone's attempts at mathematical reasoning (or anything else for that matter) is "How can I help that person improve?" Many of us have had a math teacher who seemed to have had a different goal. Please don't be one of those.